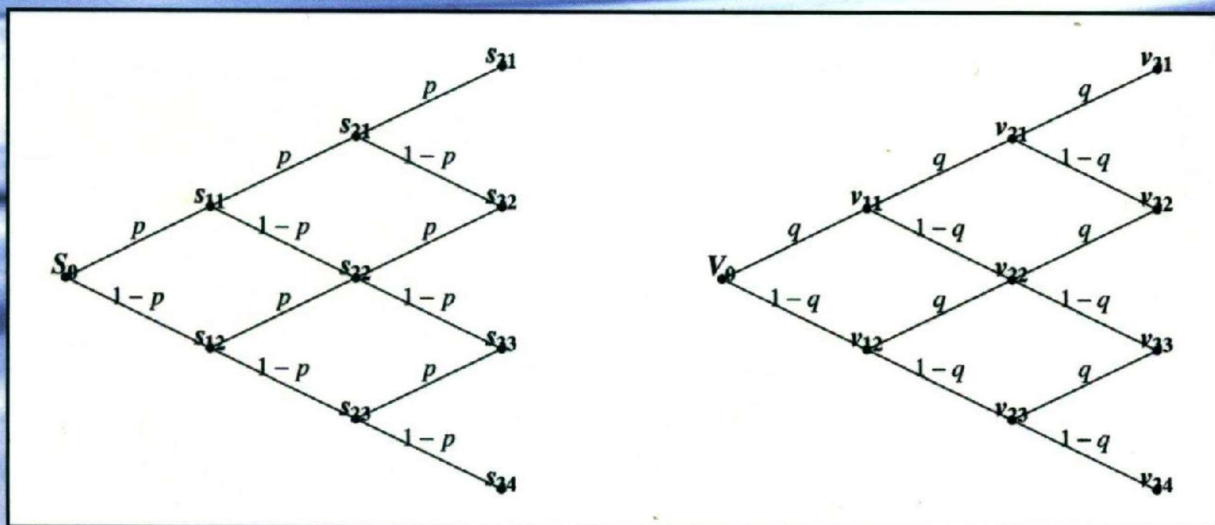


Advances in Applied Mathematics

INTRODUCTION TO FINANCIAL MATHEMATICS



Kevin J. Hastings



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*To Gay Lynn, with whom no amount of time will ever
be enough*

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